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ANALYSIS OF PATTERNS OF ATMOSPHERIC MOTIONS AT DIFFERENT SCALES BY USE OF MULTIRESOLUTION FEATURE ANALYSIS AND WAVELET DECOMPOSITION
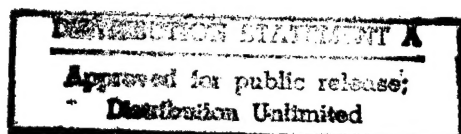
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ANALYSIS OF PATTERNS OF ATMOSPHERIC MOTIONS AT DIFFERENT SCALES BY USE OF MULTIRESOLUTION FEATURE ANALYSIS AND WAVELET DECOMPOSITION

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ABSTRACT

This report describes the work accomplished during ONR Grant No. N00014-92-J-1223, R&T Project Code 3226028-10. We have been successful in developing an entirely new technique for analyzing the form and spatial distribution of eddies in observed and simulated atmospheric flows. This technique, which we call multiresolution feature analysis (MFA) is applicable to other types of flow as well as atmospheric. To date, MFA has been used to determine the preferred patterns of motion for atmospheric flows observed over the high plains, and for a comparable large eddy simulation (LES). The technique was also used to obtain estimates of the support dimension of the turbulence for the same observed and LES flows. The results showed that the subgrid parameterization used in the LES did not capture all the intermittency of the observed flow.

Fast wavelet transform techniques have also been implemented and applied during the course of this grant. The decay of turbulence in a stratified laboratory flow was analyzed to show that mixing and restratification tend to be governed by a few major mixing events, rather than many small ones. It also showed that the major eddies at one scale do not coexist with those at other scales. This suggests that the smaller eddies in a turbulent cascade are not embedded in the larger ones, but may be attached to them.

The methods developed during the grant have been demonstrated to be workable, and have already provided some new physical insights into the nature of small scale motions. We expect to apply the analysis tools that we have developed to other flows and to modify them to provide more information about the nature of the spatial attachment of eddies at one scale to those at another. To make future applications easier, we have begun implementing and testing interpolation techniques that can be used to provide estimates of missing observations, and more regularly spaced values for the numerical results.

I OVERVIEW

A. Project Goals

The goals of the research performed under ONR Grant No. N00014-92-J-1223, R&T Project Code 3226028-10 have been to improve our understanding of atmospheric processes and the simulations of them through the application of analytical approaches derived from methods associated with multiresolution feature and wavelet analyses. We sought a clear understanding of the spatial and temporal characteristics of atmospheric motions that have many scales and are intermittent. Toward that end, we developed new tools to analyze such motions, using both field observations and numerical simulations of atmospheric motions.

The reason for pursuing the goals outlined above was in large part, because we had often observed that neither the data collected during field projects nor the outputs from elaborate numerical simulations were analyzed as effectively as we believed possible. In part, this is because field data sets are often not as complete as we would like, but that excuse does not apply to the results from numerical modeling, which are generally detailed and cover a relatively wide range of scales. Furthermore, recent improvements in computing hardware and display software now make it feasible to develop and apply the new analysis tools required to understand some of the details of atmospheric flows that affect the Navy's operations.

B. Accomplishments

Our research began with an examination of applications of fractal concepts to atmospheric flows (Ludwig 1989a, b). It was apparent that while the idea of the fractal support dimension of turbulence is useful in defining the degree to which the turbulence was space-filling, it does not provide much insight into the nature of the motions involved or the spatial relationships between turbulently active regions at different scales.

In the course of looking for methods and applications involving atmospheric fractals, we encountered the work of Jones et al. (1991) on the determination of the fractal dimension of satellite imagery using multiresolution feature analysis. One of the interesting attributes of the approach was that spatial filters could be defined and used to identify "active" areas in the images at different spatial resolutions. This allows the analyst to select patterns that have some physical significance. The method also allows the spatial distribution of the occurrence of the patterns at different scales can be preserved. The problem of the approach developed by Jones et al., was that it applied to two dimensional scalar arrays, not the three-dimensional vector arrays associated with turbulent atmospheric flows.

The next step was to extend the multi-resolution feature analysis methodology described by Jones et al. (1991) to three dimensional distributions of vectors. This turns out to be fairly straightforward conceptually, although computationally somewhat tedious. The essence of the scalar filtering used in the analysis of the imagery is to define a pattern of interest as a two-dimensional numerical array and to determine the correlation of that feature with areas in the image. The correlation is simply the sum of the products of the elements of the feature filter with those of the chosen area of the image. The same approach can be used on three-dimensional scalar fields, if the filtering pattern is also three dimensional. We developed an analogous approach for vectors by using three dimensional vector arrays as features and determining the correlation from the sum of the inner products of filtering pattern's vectors with vectors in the field being analyzed. The details of the methodology have been described by Ludwig (1993), and Ludwig and Street (1995).

The original plan was to identify idealized patterns to use as the features for analysis, but then we concluded that patterns based on the characteristics of the flows themselves would be better, particularly if they were the preferred patterns of motion in small subvolumes of the overall flow, and if the patterns were linearly independent of one another. We were able to devise a method based on empirical orthogonal functions (EOF) that could be used for identifying the preferred patterns of small scale motion and applied it to detailed Doppler radar observations of atmospheric flows (see e.g. Ludwig, 1993; Ludwig and Schneider, 1993; Ludwig et al., 1992). We then incorporated preferred small scale patterns of motion determined from the EOF analysis as filters in the multiresolution feature analysis (MFA) technique in order to study how the intensity of these patterns scales with size. This allowed us to make some inferences about the support dimension of the turbulence. The resulting MFA approach was quite successful in identifying important differences between the observed flow (Schneider, 1991) and a large eddy simulation for the same meteorological and topographical conditions (Costigan, 1992; Costigan and Cotton, 1992). These results have been reported by Ludwig (1993), and Ludwig et al. (1995, 1996).

Another successful thrust of our efforts to develop and apply new analysis techniques has been the analysis of two- and three-dimensional direct numerical simulations (DNS) using Fourier band-pass, continuous, and discrete wavelet decomposition. It was demonstrated that the wavelet decomposition techniques are very valuable for analyzing the differences between the two- and three-dimensional flows. These results were reported by Borcea et al. (1994), and led us to adapt fast wavelet transform analysis techniques (Press et al., 1992) to study the evolution of the mixing and production terms in stably stratified turbulent flow (Piccirillo et al., 1997, 1998). The new techniques were applied to an extended time series of velocity components measured during earlier laboratory experiments (Piccirillo, 1993; Piccirillo an Van Atta, 1997) to show rather conclusively that the mixing was largely the product of a limited number of large events, rather than from many smaller eddies.

We have begun developing and testing several approaches to interpolation so that we can apply the MFA techniques described above to a wider variety of observational and numerical data sets, because observations often have numerous missing values, and LES are often calculated on variably-spaced grids. Thus, we have a need for a reliable interpolation method of predictable accuracy. We have implemented several three-dimensional interpolation techniques and begun testing them.

II DATA ANALYSIS METHODS AND APPLICATIONS

A. Multiresolution Feature Analysis

The details of multiresolution feature analysis, as developed on this project for application to the study of three dimensional vector fields can be found in Ludwig (1993) or Ludwig and Street (1995). Here, we will only provide a brief outline of the development and characteristics of the methodology. We sought a method for characterizing the geometry of the highly active areas in a turbulent flow where there are intense local gradients of velocity. The approach that has evolved is particularly good for studying self-similar, intermittent atmospheric motions, but it also allows us to look at the particular patterns of motion and their spatial distribution at different scales. The technique is a variant of the multiresolution feature analysis, as described by Jones et al. (1991).

The fractal dimension D_s is often defined through its relation to the space-filling concept that arises in connection with the beta model of the turbulent cascade (see e. g. Frisch et al., 1978). It can be used as a measure of the distribution of "active" turbulent regions in space.. Jones et al. (1991) use the distribution of the peaks associated with

"features" of varying intensity to define "active" regions in satellite images. In the case of imagery, features take such forms as sharp edges between areas of different brightness, or peaks in the brightness field. The analyst is free to choose features with some physical significance. The underlying idea for the method proposed by Jones et al. (1991) is outlined below, and it serves as the starting point for the methodology that we developed. The two-dimensional analysis of a scalar field consists of the following steps:

1. Repeatedly smooth the scalar field to be analyzed (i.e. apply a low-pass filter)

2. Resample the smoothed images, taking alternate points, so that each smoothed image has half the spatial resolution and $1/4$ as many points (or $1/8$ as many, for three-dimensional analysis) as its predecessor.

3. Apply a *feature detector* (a specified pattern of local variability) repeatedly to the smoothed, resampled images from step 2.

4. Identify the peaks in the fields produced in step 3.

5. At each resolution, count the number of peaks from step 4 that exceed each of a range of threshold values.

6. Rescale the results of step 5 and plot, $(l_i)^{D_S} (n_q)_i$ versus $(l_i)^{-k} q$, where:

    k, D_S are constants producing the most nearly coincident curves for different l_i,

    l_i is the ith spatial scale, and

    $(n_q)_i$ is the number of peaks exceeding the threshold q at the ith resolution.

In a scaling field, k and D_S are the exponents that produce a set of invariant relationships, depending on the fractal properties and intermittency of the distribution of the features. If the field were purely self-similar in the usual sense, where the self similarity is independent of scale, D_S would be an integer equal to the dimension of the space containing the field -- e.g., 2 for the satellite imagery used by Jones et al. (1991) or 3 for the distribution of some scalar within a volume. However, for scale-dependent intermittency of the type associated with turbulence, where the area or volume occupied by active regions decreases at smaller scales, D_S will assume a lower fractional value. In order to estimate k and D_S, it is necessary to find the values that produce the best agreement among the curves for all scales.

The three-dimensional smoothing process for vectors is very similar to that used for scalars in two dimensions, except that the smoothing templates must be defined over a volume, rather than just an area, and they must be applied to each vector component. The three-dimensional smoothing filter that we chose weights the center point with 25 percent of the total, and distributes the remaining 75 percent among the surrounding 26 points in a 3 by 3 by 3 volume according to the inverse-squared distance from the center. While this is arbitrary, it is quite consistent with what Jones et al. (1991) did in two dimensions.

The definition of vector "features" or is somewhat subtler than the smoothing just described. We had to choose one of two possible approaches:

1. Apply the original scalar version of the technique to some scalar property of the vector field, such as divergence or the vertical component of vorticity, or

2. Define vector features and apply the multiresolution methodology directly.

Although it is somewhat more complex, we chose the second approach. In our method, we use three-dimensional arrays of vectors as the features in the analysis procedure. When this approach is taken, the feature intensity is defined as the sum of the scalar products between the vectors that are used to define the feature and the corresponding vectors in the field being analyzed. Application of the vector filters in this way produces a three-dimensional scalar array. The analysis of the resulting scalar field for its fractal properties presents the same problems as any other field of scalars, at least up to the point where the results must be interpreted. By using features defined in terms of arrays of vectors (or patterns of motion), it is possible to interpret the results in terms of specific kinds of flow patterns*. It should be noted that the two approaches can be made equivalent, if the vector features are appropriately defined to match the chosen scalar property.

At this point, we had to devise a systematic method for defining the features that would be used for the analyses. We adapted a method by which the data themselves can be used to determine what kinds of small-scale variability are important. The approach is widely used in many fields; its different variants go by various names, such as *principal component analysis, factor analysis* and *eigenvector analysis*. Following the example of Lorenz (1956), we chose to use the term *empirical orthogonal functions* (EOFs), in part because its abbreviation seems to be generally understood. Very briefly, the following steps are needed to determine the EOFs for a data set:

1. A data matrix is formed with each column containing the data (in a fixed order) for a specific, 3×3×3 subvolume of the vector field; the individual items are the differences between the observation and the average for the particular element over all the samples.

2. A new matrix (whose elements are proportional to the covariances between data paired by rows) is formed by taking the product of the data matrix and its transpose.

3. The eigenvectors of this covariance matrix are determined and ordered according to their associated amounts of explained variance.

4. The eigenvectors are normalized; the normalized eigenvectors are what we refer to as the EOFs.

The required computations are readily performed with available numerical algorithms such as those in *Numerical Recipes* (Press et al., 1992). The eigenvectors of this covariance matrix that account for the most variance in the individual patterns are used as the features in the modified multiresolution feature analysis methodology. They can be thought of as the preferred patterns of variability. These eigenvectors are the empirical orthogonal functions (EOFs) that are to be used as the features in the multiresolution feature analysis.

The peaks in the resulting feature intensity fields mark areas with the greatest local velocity variations associated with the most common motion patterns, and so they can be used to estimate the support dimension of the turbulence. The spatial distribution of the points marking the intense velocity variations at different scales would be more-or-less uniform if the turbulent cascade were space-filling*. To estimate D_s, we first identify the

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* This is reminiscent of the "whirls" that, according to Richardson (1922, page 66), feed on each other's velocities.
* Frisch et al. (1978) provide a good discussion of the relationship between the space-fillingness of the turbulent cascade and the fractal dimension of the turbulent support.

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peaks in the feature intensity field by comparing the feature intensity value at each point with those at each of the surrounding eight points to determine whether there is a local maximum or minimum. In regions where there are several contiguous points that have the same high (or low) value, only one peak is defined. After all the local maxima and minima have been identified at each of the available resolutions, the frequency with which the absolute values exceed specified thresholds is determined. This gives the values for the number of peaks whose intensities have absolute values that exceed some specified probability thresholds for each of the resolutions, and these counts can be used to estimate the scaling parameters. We used a graphical technique described by Ludwig and Street (1995) to determine the appropriate values of k and D_s.

We have used the methodology that has been developed to help us understand the details of some atmospheric flows. We have also used it to draw some conclusions about the ability of numerical simulations to reproduce the observed turbulent characteristics. These applications are discussed later.

B. Wavelet Based Techniques

The evolution and structure of homogeneous stably-stratified two-dimensional flows were investigated using direct numerical simulation. The simulation method solves the Boussinesq form of the Navier-Stokes equations using a pseudo-spectral method. The data analysis was done by using Fourier band-pass, continuous, and discrete wavelet decomposition. It was demonstrated that the wavelet decomposition techniques are very valuable for analyzing the flows.

A series of numerical experiments was performed for the two-dimensional flows and compared to equivalent three-dimensional flows. Borcea et al. (1994) found that the most important differences between the two- and three-dimensional flows were that the small scales in two dimensional flows:

1. Are more affected by viscous dissipation than the their three-dimensional counterparts.

2. Have no mechanism for gaining energy, because energy transport is toward the larger scales.

3. Decay faster than in three dimensions, leaving just the large scales, which are more affected by buoyancy forces.

All of these two-dimensional flow features were revealed by the wavelet analysis.

The initial investigations of wavelet techniques described above convinced us that there are many issues of importance to the understanding of stratified turbulence that cannot be resolved by using traditional Fourier or statistical methods. These include the effect of buoyancy on: (1) different scales of the turbulence, (2) mixing in the turbulence, and (3) production of the turbulence by mean-shear turbulence interactions. Another outstanding question was the role that large mixing and production events play in the overall physics of mixing and production in stratified turbulence. As in the multiresolution feature analysis, we were looking for new physical insights that would allow us to understand this behavior. Toward this end, we used wavelet analysis to examine these issues quantitatively. There are some aspects of the wavelet transform that make it particularly useful for these kinds of analysis.

The wavelet transform is defined as (Daubechies 1992):

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$$T_{m,n}^{wav}(f) = a_0^{-\frac{m}{2}} \int f(t)\psi(ta_0^{-m} - nb_0)dt \qquad (1)$$

where T is the transformed version of f(t), a_0 and b_0 are constants, m and n are scaling integers, and ψ is the wavelet function.

The wavelet transform is an analysis tool capable of providing information about a signal simultaneously in time and frequency or time and spatial scale (see e.g., Grossman and Morlet 1984). Traditional Fourier analysis can not provide temporal information about the analyzed signal, which limits its usefulness for elucidating physical processes. Correlation analysis, commonly used in turbulence research, also yields no temporal information. Neither does the multiresolution feature analysis. There are other types of analysis that are capable of providing simultaneous time and frequency information that have been in use for a long while (see e.g., Gabor, 1946), but the wavelet is superior to these because it makes the maximum use of the available data by increasing the spatial resolution as the frequency becomes larger, or equivalently, as the scale becomes smaller (Farge, 1992).

Liandrat and Moret-Bailly (1990) had earlier demonstrated that the wavelet had great potential for being used in turbulence research, and Farge (1992) and her coworkers (Farge, et al. 1990) had exploited the potential of wavelets to analyze turbulent fields. Some of the initial wavelet transforms used in turbulence research, while being very useful for obtaining qualitative information, were not mathematically rigorous enough to use for quantitative analysis. Daubechies (1992) and others gave the wavelet transform a better mathematical foundation that allows more quantitative analysis. Meneveau (1991) has stressed using orthonormal wavelets[*] in the study of turbulent fields. This early work and our own initial studies focused on numerical simulations, but we have extended wavelet transform analysis to data from a physical experiment, and we have used it to study buoyancy effects on stratified turbulence.

Recent work by many investigators (e.g., Holt, et al., 1992; Thoroddsen and Van Atta, 1992; Itsweire, et al., 1993 and Piccirillo and Van Atta, 1997), had raised questions about how buoyancy affects mixing in stratified turbulence. Experiments by Rohr, et al. (1988) and simulations by Holt (1990) showed that buoyancy's primary effect on the evolution of stratified turbulence in a uniform mean shear flow is to destroy correlation between vertical and streamwise components, which in turn reduces the $\overline{(uw)} \cdot dU/dz$ production term in the turbulent kinetic energy evolution equation, thereby starving the turbulence of new energy. When the buoyancy is strong enough, the turbulent kinetic energy will decrease in the downstream direction. The kinematics evident in studies by Ivey and Imberger (1991) and others have shown clearly that buoyancy influences stratified turbulence's ability to mix, but the dynamics of the process remained unknown.

Wavelet analysis seemed ideal for illuminating the results of the above studies, and for testing the conclusions drawn by Briggs (1996) and by Piccirillo and Van Atta (1997) from their studies, i.e., that mixing in stratified turbulence consists of a small number of powerful stirring events. Until we used wavelets to analyze the data, this hypothesis was based on solely on inference. We were able to establish that it is valid and to determine how increasing the buoyancy changes the behavior of the mixing in the flow. Our results are described in the next chapter.

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[*] An orthonormal wavelet function is orthogonal to itself with respect to both position (the argument of ψ in Equation (1), and dilatation $a_0^{-m/2}$ in Equation 1)

There are a number of wavelet functions φ that can be used for wavelet analysis of signals, we limited ourselves to those that are strictly orthogonal, because we wanted to analyze our data quantitatively. We chose to use a wavelet that is orthonormal to both position and dilation. We also sought an easy and fast transform algorithm, which led us to the fast wavelet transform (FWT). In the fast wavelet transform, a wavelet is chosen which, when its coefficients are rearranged, acts both as a wavelet, ψ, and as a smoothing function, ϕ, in a manner similar to multiresolution analysis (Daubechies 1992). When the signal is convolved with ϕ and ψ simultaneously, the wavelet coefficients and the smoothing for the next wavelet scale are both calculated (Press, et al. 1992). Once we decided to use orthonormal wavelets and the fast wavelet transform, we had to choose the proper wavelet basis. We used a wavelet from the Daubechies family, because they had given excellent results in earlier studies. To avoid the criticism directed against some wavelet studies that the wavelet shape had biased the results, the chosen wavelet function does not look like a "typical" turbulent eddy. Therefore, the conclusions given later about the applicability of wavelet analysis to turbulence studies, should be more general than they would have been had we selected a more eddy-like wavelet form. The 10-point Daubechies wavelet was found to have the best response to the signal, $y = a \sin(xt) + b \sin(zt)$, so it was used for the analysis described in the next chapter.

C. Interpolation Techniques

Our work during this project has involved the analysis of both observed, and simulated atmospheric motions to determine preferred small scale flow patterns and how those patterns are related to spatial scale and meteorological factors. These analyses are most easily performed when vector components are available on a regular, three-dimensional grid, without any gaps or missing data. Unfortunately, the available observations often had numerous missing values, and the large eddy simulations (LES) or Direct Numerical Simulations (DNS), while complete, were often calculated on variably-spaced grids. Thus, we have a need for a reliable interpolation method of predictable accuracy. We implemented several three-dimensional interpolation techniques. They include: (1) Inverse-distance weighting, (2) Simple linear interpolation, (3) Least-squares linear regression with inverse distance weighting of observations, (4) Least-squares polynomial (with two different polynomials) regression similar to that described by McLain (1974), also with inverse distance weighting of observations, and (5) Multiquadric interpolation (e. g., Hardy, 1990; Nuss and Titley, 1994). We have begun testing these methods by randomly selecting points in LES vector fields for a stable flow over a wavy surface (Calhoun, 1996; Calhoun and Street, 1997), and for convective, neutral, and stable boundary layers over a flat surface (Cederwall and Street, 1997). We apply the interpolation schemes with data from nearby points to estimate the vector at the selected point. We then compare the original value with the estimate for each of the methods, and determine their performance. We are also conducting similar tests on sets of radar observations of atmospheric boundary layer motions. This has been submitted for presentation at a technical meeting early next year (Ludwig et al., 1998)

III   APPLICATIONS

A.   Multiresolution Feature Analysis

We were able to apply the multiresolution feature analysis methodology developed on this project to detailed observations of the atmospheric motion that had been made with two Doppler radars during the Phoenix II field experiment in June 1984. The experiment and the reduction of the data to Cartesian coordinates, are discussed by Schneider (1991). Costigan (1992) had also done LES modeling of one of the observation periods, so we

could examine some properties of the LES that are not usually addressed, and compare the results with those obtained from the observations. The observations were made to a depth of about 2 km above a 9×9 km area about 20 km east of the Rocky Mountain Front Range foothills where the terrain is relatively flat and reasonably uniform. NOAA Wave Propagation Laboratory dual Doppler X-band radars measured the motion of chaff released from aircraft. The two radars sampled discrete, overlapping volumes to provide three-dimensional winds with 200-m spacing (in all three directions) for periods of about 20 min. The time required to sample a volume was about two minutes, so each data set included about ten individual volumes. Some of the shortcomings of these data and how they were addressed are discussed by Ludwig (1993), and Ludwig et al. (1995, 1996).

Costigan (1992) used the regional atmospheric modeling system (RAMS) developed at Colorado State University (Pielke et al., 1992; Walko et al., 1992) for an LES of conditions in the observation area corresponding to one of the observation periods. The surface characteristics and topography were matched as closely as possible to those around where the observations were made.

Richardson's (1922) poem about the cascade of turbulent energy from large to little whorls raises the following questions:

- What is the nature of the whorls in a turbulent flow?

- Do different types of whorls coexist?

- Are the whorls different for different meteorological conditions?

- To what degree are the whorls space-filling?

- Are features at one scale spatially related to those at another; if so, do the spatial relationships differ for different kinds of features?

We were able to use the MFA techniques developed on this project to obtain answers or partial answers to all but the last of the above questions. We had hoped to examine spatial relationships between features at different scales in a follow-on to this project.

We found that for the conditions that we analyzed that there are some preferred small-scale, three-dimensional patterns of motion in the atmosphere. They are not what we generally associate with conventional definitions of Richardson's "whorl." They were observed to change in response to changing meteorological conditions. Stability-related changes in patterns of motion had been observed in the at larger scales (e.g., Grossman, 1982; Williams and Hacker, 1992, 1993), which combined with our results suggests the nature of atmospheric motions changes with stability over a wide range of scales.

Even with a limited range of scales and gaps in the observations, it was possible to estimate a support dimension for both the observed data and the corresponding LES. The results that were obtained are very interesting. With continued advancements in remote sensing (e.g., Mead et al., 1998), we can expect three-dimensional data sets that will permit more detailed analysis in the future. The majority of the values of support dimension derived from data for convective conditions were between about 2.25 and 2.45, which is consistent with Chorin's (1982) numerical simulation of turbulent vortex evolution and with what Mandelbrot (1977) derived in his presentation of the beta turbulence model. Although the support dimensions estimated from a large eddy simulation were within the overall range of observed values, they were at the upper end of that range. The LES turbulence appears more space filling than was observed, but the results are inconclusive because the dimensions for the specific case simulated also tended to be high, ranging from about 2.5 to 2.8.

We concluded from our analysis that one candidate cause of the difference between observed and modeled support dimension is the parameterization of subgrid scale turbulence. If the treatment of subgrid effects distributed the energy more uniformly than does the very intermittent atmosphere, the results would be more space filling. One might expect that different subgrid models might lead to different support dimensions. While the effects of subgrid parameterization are not overly important to the simulation of convective turbulence at scales of a kilometer or larger (the primary concern of the LES studies), past results indicate that the parameterization can change the spatial distribution of feature intensities on smaller scales, so it would not be surprising if their space fillingness were also changed. An interesting numerical experiment would examine whether or not more recent subgrid scale parameterizations (e.g., Zang, 1993; Zang et al., 1993) produce different results.

B. Wavelet Based Techniques

As noted earlier, the 10-point Daubechies wavelet had the best response to the signal, y=a sin(xt) + b sin(zt), so we used it to analyze a time signal from Piccirillo's (1993) laboratory data. The resulting power spectrum for the streamwise velocity was calculated and compared with the power spectrum obtained from Fourier techniques, as were the temperature-vertical velocity cospectra. The results obtained from wavelet analysis were found to be nearly identical to those obtained from Fourier analysis (Piccirillo, et al., 1997 and 1998), so we felt confident in proceeding to use wavelet analysis to extract higher order moments, such as scale-based skewness (the third order moment), and scale-based intermittency (the fourth order moment) at each scale, in addition to the usual second order moments from an extended time series of observations of uniform mean-shear, stably-stratified turbulence reported by Piccirillo (1993). The results of the original, more conventional analysis of these data was reported by Piccirillo and Van Atta (1997). The Reynolds number based on the integral scale of the turbulence was approximately 150 in these experiments. Fluctuating velocities u and w and fluctuating temperature θ were measured at a 5 kHz rate at nineteen locations from 0.30 to 5. 18 m downstream of the turbulence generating grid. The analysis was based on approximately 3.5 minutes of observations, which we refer to as a megasample (=1,048,576 individual observations). Five different flow regimes were analyzed:

1. A neutral or unstratified flow (Ri=0)

2. A slightly stratified flow (Ri=0.04), where the turbulent kinetic energy increased downstream.

3. A stratified flow (Ri =0.19) where the turbulent kinetic energy is stationary downstream.

4. A heavily stratified flow (Ri=0.44) where the turbulent kinetic energy decreases downstream.

5. The same heavily stratified flow as number 4 with turbulent decreasing downstream kinetic energy *and* a small scale flux reversal of temperature (see Piccirillo and Van Atta 1997 for details).

To study the scale based intermittency (not available from Fourier-based analysis), we treated the wavelet coefficients at a particular scale as a time signal, computed the intermittency (or kurtosis) of that time signal, and then compared the results across scales from 6 mm to 6 m. According to Meneveau (1991), if the process that produced the wavelet coefficients is Gaussian (kurtosis = 3), then the wavelet coefficients themselves should have a Gaussian distribution. Our results for the streamwise velocity fluctuation u,

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the vertical fluctuation w, and the temperature fluctuation θ are very similar to those found by Meneveau (1991), i.e., the intermittency for both the velocity and temperature fluctuations is close to 3.0 for most of the range of scales that we would define as being turbulent. However, at smaller scales (higher frequency), the intermittency rises as the smallest scale of the turbulence is approached, and then falls back towards 3, showing that influence of viscosity causes the smaller scale turbulence to be more intermittent .

Our analyses found no clear effect from increasing the buoyancy forces in the flow, nor between the either of the velocity component results and the temperature, so the scale-based intermittency can be used to determine what scales are truly turbulent. The scale-based intermittency results for the buoyancy flux correlation $\overline{w\theta}$ and the production term correlation $\overline{uw}$ behave very differently from the individual components. For the least stratified case, both $\overline{w\theta}$ and $\overline{uw}$ scale-based intermittency are around 6 for frequencies less that 10 Hz (scale=0.19 m), and increase steadily to a maximum of about 4,000 at 300 Hz (scale=6 mm) for $\overline{w\theta}$, and 600 at 80 Hz (scale = 24 mm) for $\overline{uw}$. Scale-based intermittency of $\overline{w\theta}$ increases as the buoyancy forces increase until we reach the case where small-scale flux reversal occurs, and the scale-based intermittency decreases dramatically. Even when there is little buoyancy, overall mixing depends on powerful stirring events. The increase in scale-based $\overline{w\theta}$ intermittency that accompanies increased buoyancy suggests that the number of powerful stirring events decreases until the flux changes sign and the number of stirring events again increases. However, these events are now restratifying events, not stirring (active mixing) events.

Our analyses showed that when a small amount of stratification is added, the behavior of the scale-based $\overline{uw}$ intermittency remains unchanged. Only when the buoyancy forces become strong enough to change the qualitative behavior of the turbulence does the scale-based intermittency change and become an increasing function of the buoyancy, reaching values as high at 30,000. Our results showed that even in the unstratified case, the $\overline{uw}$ correlation is somewhat dependent on a few large coefficients, which physically corresponds to the production being dominated by a few powerful events. As the buoyancy forces increase, one can postulate (based on the increasing intermittency) that there are still production events, but fewer of them. Holt, et al. (1992) found that for small values of Ri, the spectral behavior of the turbulence changed dramatically, but this was not the case with our $\overline{uw}$ results.

An analysis of the probability distribution functions (PDF) of the wavelet coefficients showed that scale invariance exists for the larger scales of the turbulence, and the turbulence becomes non-invariant only at the smaller scales, where the intermittency becomes much larger. The PDFs also suggest that the physics of mixing is not scale-dependent (i.e. mixing events at different scales are not different from one another), but rather that changes in the overall behavior of the mixing in the flow are simply due a difference in the relative number of stirring and restratifying events which occur, and that the overall state of the turbulence, either stirring or restratifying, depends on the number of strong outlying events that occur. Strongly skewed large and intermediate scale results showed the strength of the production at those scales. At smaller scales, it was clear that only a small relative amount of production occurred. As the stratification is increased, the small and intermediate scale results become less skewed, followed by the large scale results, showing clearly that the buoyancy forces are affecting the smaller scales of the turbulent production more strongly than the larger scales, but the large scale results are also affected, so that when the physics of the turbulence is dominated by buoyancy, there is little or no production at any scale. Our findings in this regard agree with those of Holt, et al. (1992). These results suggest that, as with mixing, changes in the overall behavior of

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the production are due to differences in the relative number of large production and destruction events occurring in the turbulence.

In an effort to understand the physics of the mixing and production better, we located and tabulated all the points where the value of the $w\theta$ and uw products (not the integrated values, $\overline{w\theta}$ and $\overline{uw}$) were more than two standard deviations above or below the mean. The results showed that the sign of $\overline{w\theta}$ and $\overline{uw}$ were determined by the difference between the number of the extreme events that were above the mean minus the number below it. Although the $w\theta$ outliers represented only about 3-4% of the total number of points, they accounted for 32 to 50% of the total $\overline{w\theta}$ correlation. In the case of extreme uw, the extreme events were less than five percent of the total, but accounted for up to half the total turbulence production. These results show the immense importance, even for the least stratified case, that the strong mixing events had on the overall mixing. It was found that the percentage of stirring events was a decreasing function of scale for all the different stratifications, suggesting that buoyancy affects the small scale mixing physics even at very low Ri. It was also found that restratifying events, while rare, occurred at all scales for the weakest stratification.

Another very interesting finding of our study was that the existence of large mixing events at one scale precluded mixing at other scales, suggesting that mixing events might be temporally linked, so that mixing events at one scale are then followed in time by events at a different scale. We computed the lagged correlation coefficients between different scales and found that almost all of them were between -0.1 and +0.1, meaning that if any temporal linking exists between different scales, it must be very weak.

C. Interpolation Techniques

We have been analyzing observed and simulated atmospheric motions to determine preferred small scale flow patterns and how those patterns are related to spatial scale and meteorological factors. These analyses and others that are planned to determine relationships between the patterns at different scales are most easily performed when vector components are available on a regular, three-dimensional grid, without any gaps or missing data. Unfortunately, the observations often have numerous missing values and LES or DNS results, while complete are often calculated on variably-spaced grids. Thus, we have a need for a reliable interpolation method of predictable accuracy. Similar problems arise in nested grid models, and many areas of data analysis, so our results will have applicability that extends beyond the MFA and wavelet methodologies described here.

IV   ACCOMPLISHMENTS

A. Publications

Ludwig, F. L., 1989a: "Atmospheric Fractals -- A Review," Environ. Software, 4, 9-16.

Ludwig, F. L., 1989b: "Fractal Concepts and the Analysis of Atmospheric Inhomogenieties," in Air Pollution Modeling and Its Application VII," (ed.: H. van Dop), Plenum Pub., New York, pp. 451-460.

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Piccirillo, P., R. L. Street and J. R. Koseff, 1998: " Wavelet Analysis of Uniform Mean-Shear Stably-Stratified Flow," submitted to Phys. of Fluids, referees' comments currently being addressed.

B. Presentations

Calhoun, R. 1996: "Turbulent Flow over Sinusoidal Topography," presented at Fall Meet. Amer. Geophys. Union, Paper A7IA-12, San Francisco, Dec. 15-19.

Calhoun, R. J. and R. L. Street, 1997: "A Comparison of Turbulent Flow over Two- and Three-Dimensional Sinusoidal Topographies", Preprints 12th Symp. Bound. Layers and Turbulence, Vancouver, B. C., July 27-August 1, Amer. Meteorol. Soc., Boston, pp. 215-216.

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C. Degrees Awarded

Ludwig, F. L., 1993: Ph.D. degree in Atmospheric Sciences, Dept. of Civil Engineering, Stanford University, Stanford, CA.

D. Laboratory Reports

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V CONCLUSIONS

Both the wavelet and the MFA methods were applied to existing data sets that had been previously analyzed by other methods. We feel that the success that has been achieved illustrates how valuable these new approaches and new tools are. We have produced methods that can be applied to existing observations and simulations to gain new insights into the physics of turbulent motions. The use of existing data sets is certainly more cost effective than the undertaking of major new experimental efforts. There are many numerical simulations available (e.g., Yuan, 1997; Yuan and Street, 1996, 1997; Calhoun, 1996; Calhoun and Street, 1997; Cederwall and Street, 1997). New instrumentation (e.g., Mead et al., 1998) is providing new observational data sets also.

The MFA analysis methods developed during the course of this grant have been demonstrated capable of determining preferred patterns of small scale motion in a flow, and have shown that the form of those patterns depends on the stability of the flow. The methods have provided estimates of the support dimension in observed and simulated flow, and suggested that the closure scheme used in the LES results that were studied does not reproduce the observed intermittency of a similar observed flow. These methods should in the future provide a means by which we may be able to study the details of the relationship between turbulent regions at different scales, both regard to location, and the patterns of motion.

Wavelet analysis of a turbulent flow with the fast wavelet transform has proven to be both useful and relatively straightforward. The 10-point Daubechies wavelet that we used is not at all optimized for the study of turbulent eddies, but it still gave excellent results, which suggests that wavelet analysis is a very good tool for studying turbulence. The physical insights gained from the wavelet analysis have helped us understand the physics of turbulent mixing and production by demonstrating that turbulent mixing consists of a preponderance of strong events, both stirring and restratifying. These events consist of simple eddies at a single scale. That means that, although there are eddies whose sizes cover a wide range of scales, they do not necessarily overlap in space. This finding is interesting because it seems to answer a question raised by Frisch et al. (1978) in their description of the beta cascade model of turbulence. Like most cascade models of turbulence, they do not specify any spatial relationship between eddies of different sizes, stating no more than that the smaller eddies may be related to the larger ones either by embedding or by attachment. The distinction between embedding and attachment, and the relative locations at the different scales are particularly important questions from a practical standpoint, because a definitive answer might allow statistically reasonable extrapolation of motion patterns to smaller scales. Our wavelet results suggest attachment rather than embedding. The obvious next step is to modify the MFA procedure so that we can determine if there are preferred attachment geometries.

The wavelet analyses also showed that buoyancy affects the overall mixing by changing the relative numbers of strong events which occur, while affecting the form of each individual event very little. Production results are generally similar, but buoyancy seems to affect both the number and strength of the strong production events, while the production events themselves seem to be very similar in structure to the simple eddies found for mixing. This wavelet finding is also amenable to testing with MFA, because MFA provides a method for characterizing the form of the eddies in three dimensions and for counting their density under different stability conditions.

Obviously, there are many exciting new applications of the analysis tools developed during the course of this grant. We have noted that large open regions of missing observations hamper analysis. The unevenly spaced grids of some simulations present related, but much less serious difficulties. It is for these reasons that we have begun developing and testing methods for estimating missing values or recasting data to more easily managed coordinates. While this effort is still in its early stages, the results are quite encouraging. When we have completed our interpolation studies, we will be able to attack the very interesting problems outlined in the preceding paragraphs.

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